

**Algebra: Simplifying Algebraic Expressions, Expanding Brackets,
Solving Linear Equations, Applications. KS3, KS4. Non-Calculator.**

A. Simplify the following expressions:

1. $5a + 3a$

2. $6a - 4a$

3. $4a + a$

4. $x + x + x + x$

5. $a - a$

6. $3a + 2a - 5a$

7. $3a + 5c - a + 2c$

8. $3x + 2x + 3y - y$

9. $3x - x + 3 - 2$

10. $3x + y - x + 4y$

11. $3x + 2y - 3x + 4y$

12. $2x + 5y - 3y + x$

13. $p + q - p - q$

14. $p + q + p + q + p$

15. $4p - 5p$

16. $5c + 2d - 3c - 4d$

17. $5x - 3y + 2x - 4y$

18. $5p - 3q + 2 - 4p + 5 + 4q$

19. $2ac + 3ac - 4ac$

20. $xy + yx$

21. $2xy - 4ac + 5yx + 4ac$

22. $3xy + 4xy - xy$

23. $3cd - 4cd + cd$

24. $xy + yx - 2xy + 1$

25. $2ab + 3cd - 4ab - 3cd$

26. $4y^2 - 3y^2$

27. $4x^3 - x^3$

28. $3x^2 + y^3 - x^2 - y^3$

29. $4y^2 + 5y - 3y^2 - 4y$

30. $2x^2 + 3x - 5x^2 - x + 8$

31. $x^2 + x^2 + x^2$

32. $x^2 + x^2 + 3x^2$

33. $x^2 + x^2 - x^2$

34. $x^2y + xy + x^2y$

35. $x^2y + xy^2 - x^2y + 2y^2x$

B. Expand the brackets and simplify where possible.

1. $4(x - 3)$

2. $4(2x - 3)$

3. $2(3 - 4y)$

4. $x(x + 1)$

5. $x(x - 2)$

6. $x(x^2 + 4x - 3)$

7. $y(x - y^2)$

8. $4(p + 2) + 3(2p - 3)$

9. $2(3p + 2) + 3(2p - 3)$

10. $3(2p - 5) + 2(3p - 3)$

11. $2p(p + 2) + 3p(2p - 3)$

12. $3p(p - 2) + 2p(3p - 2)$

13. $2p(p - 3) + 3p(3p - 2)$

14. $x(x^2 - 2y) + 3x^2(x + 2y)$

15. $-(x - 3)$

16. $-4(2x - 3)$

17. $-2(3 - 4y)$

18. $-x(x + 1)$

19. $-x(x - 2)$

20. $-x(x^2 + 4x - 3)$

21. $-y(x - y^2)$

22. $7(p + 2) - 3(2p - 3)$

23. $2(3p + 2) - 3(2p - 3)$

24. $3(2p - 5) - 2(3p - 3)$

25. $2p(p + 2) - 3p(2p - 3)$

26. $3p(p - 2) - 2p(3p - 2)$

27. $2p(p - 3) - 3p(3p - 2)$

28. $3(x - 2y) - 2(x - 3y)$

29. $2(3x + 1) - 5(2x - 3)$

30. $x(x^2 - 2y) - 3x^2(x + 2y)$

31. $2(3x + 1) - (2x - 3)$

32. $2(p - 4) + 3(2p - 1)$

33. $a(a + 2b - 3c) + 3c(a - 2b + 3c) - 2b(a - b - 3c)$

34. $a(b - c + d) - a(b - c + d)$

35. $3a(2b - 3c + 4d) - 2a(3b - c + 6d)$

36. $5 - 2(x - 3)$

37. $6 + 4(3 - x)$

38. $6 + (2x + 6)$

39. $6 - (2x + 6)$

40. $2x^2(4xy - 5) - 8yx^3 + 9x^2$

C. Solve the following equations:

1. $x + 3 = 9$

2. $2x = 6$

3. $4 - x = 5$

4. $2x + 3 = 13$

5. $2x = 1$

6. $3x = 2$

7. $4x = 20$

8. $4x - 1 = 19$

9. $4x = -20$

10. $2x = -6$

11. $4x = -8$

12. $4x = -1$

13. $2x + 3 = -5$

14. $2x - 3 = 5$

15. $2x - 3 = x + 2$

16. $7x - 3 = 2x + 12$

17. $7y - 8 = 5y + 2$

18. $4x + 5 = 2x - 11$

19. $5x - 6 = 2x - 15$

20. $x + 2x = -15$

21. $3x - 5 = 4x - 7$

22. $2x + 7 = 5x - 3$

23. $2x + 7 = 12 - 3x$

24. $6y - 2 = 8y - 5$

25. $8 - 4x = 10 - 2x$

26. $12 = 3x - 6$

27. $3(x - 5) = 12$

28. $5(2x - 3) = 15$

29. $5(3 - 2x) = 30$

30. $3(2x - 4) = 8$

31. $7x + 2 = 5(x - 2)$

32. $22 - 3x = 2(x + 6)$

33. $13 - 3x = 4(x - 2)$

34. $x - 18 = 2(2x - 3)$

35. $4(2x - 3) = 3x - 27$

36. $3(2x - 5) = 6 + 2(x - 3)$

37. $4 - (3x - 5) = 6 - (2x + 7)$

38. $x(x + 5) = x^2 - 15$

39. $3x(2 + x) = x(3x - 2) - 24$

40. $3(x - 4) - 2(x - 5) = 6x - 2(x - 5)$

APPLICATIONS:

1. The width of a rectangle is x centimeters and its length is $(x + 5)$ cm.

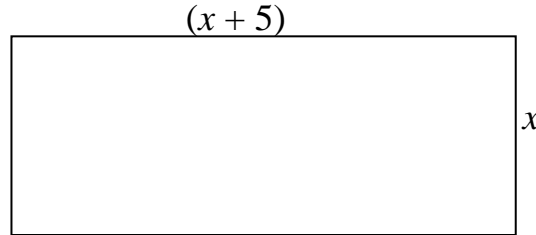


Diagram not drawn to scale

- (a) Write down an expression for the perimeter of the rectangle, giving your answer in its simplest form.

The perimeter of the rectangle is 62 cm.

- (b) Work out the length of the rectangle.

2. The diagram below is a rectangle. All measurements are in centimeters.

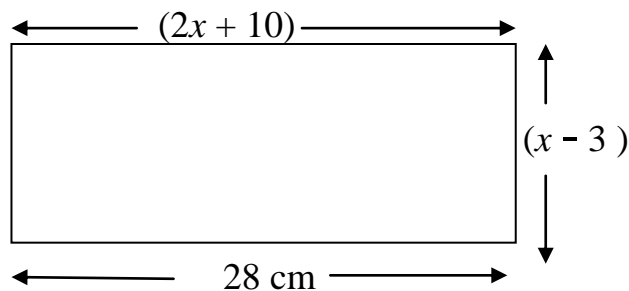


Diagram not drawn to scale

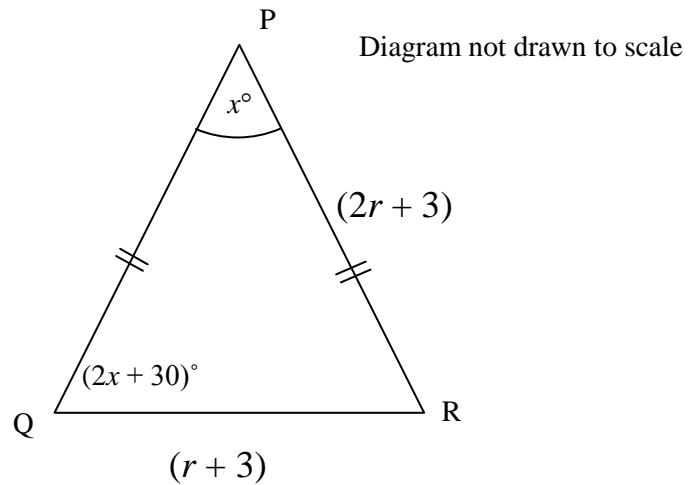
- (a) Work out the value of x .
(b) Hence, work out the perimeter and area of the rectangle.

3. PQR is an isosceles triangle with $PQ = PR$, and angle $QPR = x^\circ$.

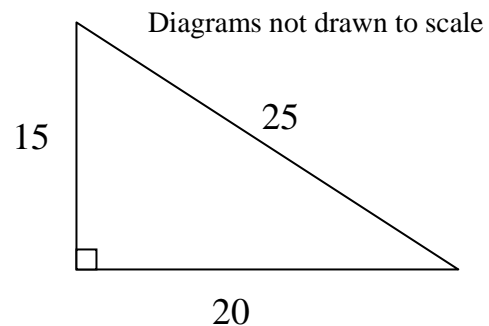
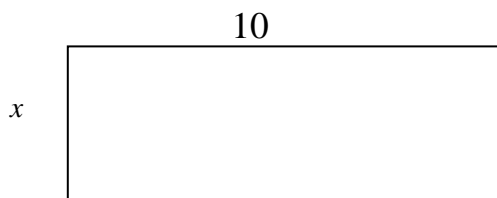
$$\text{Angle } PQR = (2x + 30)^\circ$$

$$PR = (2r + 3) \text{ cm}$$

$$QR = (r + 3) \text{ cm}$$



- (a) Find an expression for the perimeter of the triangle in terms of r , giving your answer in its simplest form.
- (b) Work out the value of r , if the perimeter is 49 cm.
- (c) Work out the value of x .
4. The area of the right-angled triangle is equal to the area of the rectangle. Work out the value of x . All measurements are in centimeters.



ANSWERS/SOLUTIONS

- A.
- | | |
|----------------|--------------------------------|
| ① <u>8a</u> | ②② <u>6xy</u> |
| ② <u>2a</u> | ②③ <u>0</u> |
| ③ <u>5a</u> | ②④ <u>1</u> |
| ④ <u>4x</u> | ②⑤ <u>-2ab</u> |
| ⑤ <u>0</u> | ②⑥ <u>y²</u> |
| ⑥ <u>0</u> | ②⑦ <u>3x³</u> |
| ⑦ <u>2a+7c</u> | ②⑧ <u>2x²</u> |
| ⑧ <u>5x+2y</u> | ②⑨ <u>y²+y</u> |
| ⑨ <u>2x+1</u> | ③⑩ <u>-3x²+2x+8</u> |
| ⑩ <u>2x+5y</u> | ③① <u>3x²</u> |
| ⑪ <u>6y</u> | ③② <u>5x²</u> |
| ⑫ <u>3x+2y</u> | ③③ <u>x²</u> |
| ⑬ <u>0</u> | ③④ <u>2x²y + xy</u> |
| ⑭ <u>3p+2q</u> | ③⑤ <u>3xy²</u> |
| ⑮ <u>-p</u> | |
| ⑯ <u>2c-2d</u> | |
| ⑰ <u>7x-7y</u> | |
| ⑱ <u>p+q+7</u> | |
| ⑲ <u>ac</u> | |
| ⑳ <u>2xy</u> | |
| ㉑ <u>7xy</u> | |

B.

① $4x-12$ ② $8x-12$ ③ $6-8y$ ④ x^2+x

⑤ x^2-2x ⑥ x^3+4x^2-3x ⑦ $xy-y^3$

⑧ $4p+8+6p-9$
 $= 10p-1$ ⑨ $6p+4+6p-9$
 $= 12p-5$

⑩ $6p-15+6p-6$
 $= 12p-21$ ⑪ $2p^2+4p+6p^2-9p$
 $= 8p^2-5p$

⑫ $3p^2-6p+6p^2-4p$
 $= 9p^2-10p$ ⑬ $2p^2-6p+9p^2-6p$
 $= 11p^2-12p$

⑭ $x^3-2xy+3x^3+6x^2y$
 $= 4x^3-2xy+6x^2y$ ⑮ $-x+3x$

⑯ $-8x+12$ ⑰ $-6+8y$ ⑱ $-x^2-x$

⑲ $-x^2+2x$ ⑳ $-x^3-4x^2+3x$

㉑ $-xy+y^3$ ㉒ $7p+14-6p+9$
 $= p+23$

㉓ $6p+4-6p+9$
 $= 13$ ㉔ $6p-15-6p+6$
 $= -9$

㉕ $2p^2+4p-6p^2+9p$
 $= -4p^2+13p$ ㉖ $3p^2-6p-6p^2+4p$
 $= -3p^2-2p$

㉗ $2p^2-6p-9p^2+6p$
 $= -7p^2$ ㉘ $3x-6y-2x+6y$
 $= x$

$$\begin{aligned} (29) \quad & 6x + 2 - 10x + 15 \\ & = \underline{\underline{-4x + 17}} \end{aligned}$$

$$\begin{aligned} (30) \quad & x^3 - 2xy - 3x^3 - 6x^2y \\ & = \underline{\underline{-2x^3 - 2xy - 6x^2y}} \end{aligned}$$

$$\begin{aligned} (31) \quad & 6x + 2 - 2x + 3 \\ & = \underline{\underline{4x + 5}} \end{aligned}$$

$$\begin{aligned} (32) \quad & 2p - 8 + 6p - 3 \\ & = \underline{\underline{8p - 11}} \end{aligned}$$

$$\begin{aligned} (33) \quad & a^2 + \cancel{2ab} - \cancel{3ac} + \cancel{3ca} - \cancel{6cb} + 9c^2 - \cancel{2ba} + \cancel{2b^2} + \cancel{6bc} \\ & = \underline{\underline{a^2 + 9c^2 + 2b^2}} \end{aligned}$$

$$\begin{aligned} (34) \quad & \cancel{ab} - \cancel{ac} + \cancel{ad} - \cancel{ab} + \cancel{ac} - \cancel{ad} \text{ OR } \begin{array}{l} \text{let } b - c + d = x \\ ax - ax = 0 \end{array} \\ & = \underline{\underline{0}} \end{aligned}$$

$$\begin{aligned} (35) \quad & \cancel{6ab} - 9ac + \cancel{12ad} - \cancel{6ab} + 2ac - \cancel{12ad} \\ & = \underline{\underline{-7ac}} \end{aligned}$$

$$\begin{aligned} (36) \quad & 5 - 2x + 6 \\ & = \underline{\underline{11 - 2x}} \end{aligned}$$

$$\begin{aligned} (37) \quad & 6 + 12 - 4x \\ & = \underline{\underline{18 - 4x}} \end{aligned}$$

$$\begin{aligned} (38) \quad & 6 + 2x + 6 \\ & = \underline{\underline{12 + 2x}} \end{aligned}$$

$$\begin{aligned} (39) \quad & 6 - 2x - 6 \\ & = \underline{\underline{-2x}} \end{aligned}$$

$$\begin{aligned} (40) \quad & \cancel{8x^3y} - 10x^2 - \cancel{8yx^3} + 9x^2 \\ & = \underline{\underline{-x^2}} \end{aligned}$$

C.

$$\textcircled{1} \quad x = 9 - 3 = \underline{\underline{6}}$$

$$\textcircled{2} \quad x = \frac{6}{2} = \underline{\underline{3}}$$

$$\textcircled{3} \quad 4 - 5 = x \\ \underline{\underline{-1 = x}}$$

$$\textcircled{4} \quad 2x = 13 - 3 \\ 2x = 10 \\ \underline{\underline{x = 5}}$$

$$\textcircled{5} \quad x = \underline{\underline{\frac{1}{2}}}$$

$$\textcircled{6} \quad x = \underline{\underline{\frac{2}{3}}}$$

$$\textcircled{7} \quad \underline{\underline{x = 5}}$$

$$\textcircled{8} \quad 4x = 20 \\ \underline{\underline{x = 5}}$$

$$\textcircled{9} \quad \underline{\underline{x = -5}}$$

$$\textcircled{10} \quad \underline{\underline{x = -3}}$$

$$\textcircled{11} \quad \underline{\underline{x = -2}}$$

$$\textcircled{12} \quad x = \underline{\underline{-\frac{1}{4}}}$$

$$\textcircled{13} \quad 2x = -8 \\ \underline{\underline{x = -4}}$$

$$\textcircled{14} \quad 2x = 8 \\ \underline{\underline{x = 4}}$$

$$\textcircled{15} \quad 2x - x = 2 + 3 \\ \underline{\underline{x = 5}}$$

$$\textcircled{16} \quad 7x - 2x = 12 + 3 \\ 5x = 15 \\ \underline{\underline{x = 3}}$$

$$\textcircled{17} \quad 7y - 5y = 2 + 8 \\ 2y = 10 \\ \underline{\underline{y = 5}}$$

$$\textcircled{18} \quad 4x - 2x = -11 - 5 \\ 2x = -16 \\ \underline{\underline{x = -8}}$$

$$\textcircled{19} \quad 5x - 2x = -15 + 6 \\ 3x = -9 \\ \underline{\underline{x = -3}}$$

$$\textcircled{20} \quad 3x = -15 \\ \underline{\underline{x = -5}}$$

$$\textcircled{21} \quad -5 + 7 = 4x - 3x \\ \underline{\underline{2 = x}}$$

$$\textcircled{22} \quad 7 + 3 = 5x - 2x \\ 10 = 3x \\ \underline{\underline{\frac{10}{3} = x}} = \underline{\underline{\frac{10}{3}}} \text{ both acceptable}$$

$$\begin{aligned} (23) \quad 2x + 3x &= 12 - 7 \\ 5x &= 5 \\ \underline{x} &= \underline{1} \end{aligned}$$

$$\begin{aligned} (24) \quad -2 + 5 &= 8y - 6y \\ 3 &= 2y \\ \underline{1.5} &= \underline{\frac{3}{2}} = \underline{y} \end{aligned}$$

$$\begin{aligned} (25) \quad 8 - 10 &= -2x + 4x \\ -2 &= 2x \\ \underline{-1} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (26) \quad 12 + 6 &= 3x \\ 18 &= 3x \\ \underline{6} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (27) \quad 3x - 15 &= 12 \quad \text{OR} \quad (x - 5) = \frac{12}{3} = 4 \\ 3x &= 27 & x &= 4 + 5 = 9 \\ \underline{x} &= \underline{9} & & \end{aligned}$$

$$\begin{aligned} (28) \quad 10x - 15 &= 15 \quad \text{OR} \quad 2x - 3 = \frac{15}{5} = 3 \\ 10x &= 30 & 2x &= 6 \\ \underline{x} &= \underline{3} & \underline{x} &= \underline{3} \end{aligned}$$

$$\begin{aligned} (29) \quad 15 - 10x &= 30 \quad \text{OR} \quad 3 - 2x = \frac{30}{5} = 6 \\ 15 - 30 &= 10x & 3 - 6 &= 2x \\ -15 &= 10x & -3 &= 2x \\ \underline{-1.5} &= \underline{-\frac{15}{10}} = \underline{x} & \underline{-\frac{3}{2}} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (30) \quad 6x - 12 &= 8 \\ 6x &= 20 \\ \underline{x} &= \underline{\frac{20}{6}} = \underline{\frac{10}{3}} = \underline{3\frac{1}{3}} \end{aligned}$$

$$\begin{aligned} (31) \quad 7x + 2 &= 5x - 10 \\ 7x - 5x &= -10 - 2 \\ 2x &= -12 \\ \underline{x} &= \underline{-6} \end{aligned}$$

$$\begin{aligned} (32) \quad 22 - 3x &= 2x + 12 \\ 22 - 12 &= 2x + 3x \\ 10 &= 5x \\ \underline{2} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (33) \quad 13 - 3x &= 4x - 8 \\ 13 + 8 &= 4x + 3x \\ 21 &= 7x \\ \underline{3} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (34) \quad x - 18 &= 2(2x - 3) \\ x - 18 &= 4x - 6 \\ -18 + 6 &= 4x - x \\ -12 &= 3x \\ \underline{-4} &= \underline{x} \end{aligned}$$

$$\begin{aligned} (35) \quad 8x - 12 &= 3x - 27 \\ 8x - 3x &= -27 + 12 \\ 5x &= -15 \\ \underline{x} &= \underline{-3} \end{aligned}$$

$$\begin{aligned} (36) \quad 6x - 15 &= 6 + 2x - 6 \\ 6x - 2x &= 15 \\ 4x &= 15 \\ x &= \frac{15}{4} \text{ OR } 3\frac{3}{4} \end{aligned}$$

$$\begin{aligned} (37) \quad 4 - 3x + 5 &= 6 - 2x - 7 \\ 4 + 5 - 6 + 7 &= -2x + 3x \\ 10 &= x \end{aligned}$$

$$\begin{aligned} (38) \quad x^2 + 5x &= x^2 - 15 \\ x^2 + 5x - x^2 &= -15 \\ 5x &= -15 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} (39) \quad 6x + 3x^2 &= 3x^2 - 2x - 24 \\ 6x + 3x^2 - 3x^2 + 2x &= -24 \\ 8x &= -24 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} (40) \quad 3x - 12 - 2x + 10 &= 6x - 2x + 10 \\ x - 2 &= 4x + 10 \\ -2 - 10 &= 4x - x \\ -12 &= 3x \\ -4 &= x \end{aligned}$$

Applications

$$\begin{aligned} (1) \quad (a) \quad x + x + 5 + x + x + 5 & \text{ (OR) } 2(x + x + 5) \\ &= 4x + 10 &= 2(2x + 5) \\ & &= 4x + 10 \end{aligned}$$

$$\begin{aligned} (b) \quad 4x + 10 &= 62 \\ 4x &= 52 \\ x &= 13 \end{aligned}$$

$$\begin{aligned} \text{OR half the perimeter} &= x + x + 5 = 31 \\ 2x &= 26 \\ x &= 13 \end{aligned}$$

The Length = $x + 5 = 13 + 5 = 18\text{cm}$

$$\begin{aligned} \textcircled{2} \text{ (a)} \quad 2x+10 &= 28 \quad \text{opposite sides equal.} \\ 2x &= 18 \\ x &= \underline{\underline{9}} \end{aligned}$$

$$\text{(b)} \quad \text{Hence the width} = x - 3 = 9 - 3 = 6 \text{ cm.}$$

$$L = 28, W = 6, \quad \text{Perimeter} = 2(28 + 6) = 2(34) = 68 \text{ cm}$$

$$\text{Area} = L \times W = 28 \times 6 = 168 \text{ cm}^2$$

$$\begin{aligned} \textcircled{3} \text{ (a)} \quad \text{Perimeter} &= 2r+3 + 2r+3 + r+3 \\ &= \underline{\underline{5r+9}} \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 5r+9 &= 49 \\ 5r &= 40 \\ r &= \underline{\underline{8}} \end{aligned}$$

$$\text{(c)} \quad \text{angle } \hat{P}\hat{R}\hat{Q} = 2x+30 \quad \text{isosceles triangle.}$$

$$\text{Sum of all 3 angles} = 180$$

$$2x+30 + 2x+30 + x = 180$$

$$5x + 60 = 180$$

$$5x = 120$$

$$x = \frac{120}{5} = \frac{240}{10} = \underline{\underline{24}}$$

$$\begin{aligned} \textcircled{4} \quad \text{Area of triangle} &= \frac{1}{2} \times b \times h = \frac{1}{2} \times 20 \times 15 = 150 \\ \text{area of rectangle} &= 10x \end{aligned}$$

$$10x = 150$$

$$x = \frac{150}{10} = \underline{\underline{15}}$$

I hope you find this useful. If you find any errors, please let me know.